Stability of Centrally Loaded Glass Columns

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ABSTRACT

Although glass can be applied in load bearing structures due to the development of glass strengthening procedures, it remains still fragile. Glass walls can be supported by glass fins against wind load, as well as a slab can be supported by glass columns. However the appropriate question is where can be found the limit of the load bearing capacity of the glass columns. This paper focuses on the stability issues. More than 120 scaled-size specimens were loaded under compression to study the buckling behaviour of glass columns with flat shaped cross-section. Laboratory experiments were carried out at the BME, Department of Construction Materials and Technologies. Laminated glass consisted of different glass layers (e.g. variable thicknesses, type of glass layers etc.) were compressed by concentrated load. The loading force and displacements were measured. Increment method was applied to study and determine the Serviceability Limit State (SLS) based on horizontal displacement results. The experimental results were studied to find the relationship between the type of buckling and the glass columns. Based on laboratory experiments and theoretical calculations the influencing factors on critical force and buckling behaviour of glass columns were studied. The authors draw attention to the difficulties of design of glass columns.

Keywords: glass column, buckling, load bearing glass, stability, transparency, compression

INTRODUCTION

Glass is called also the material of the third millennium. Although glass is a brittle material, its brittleness has been a well-known property alongside its transparency (Pankhardt et al. 2012 – [8]). This paper focuses on the buckling behaviour of the load bearing glass elements, especially glass columns. Laboratory tests were carried on axially compressed glasses in the BME, Department of Construction Material and
Technologies. Based on the laboratory experimental results, influence of several physical properties of glass columns were compared to each other e.g. effect of the rate of loading, heat strengthening, effect of the different height of the specimen and the effect of the lamination with total thickness of 12 mm (single layer glasses, and laminated glasses consist two, and three layers) on the behaviour of glass columns. Three different stages were determined and introduced in the buckling behaviour of glass columns. The critical buckling force was studied based on the international theoretical results and the laboratory tests (Nehme and Jakab and Nehme 2013 - [6]).

Continuing the previous laboratory research, the stability and design methods of glass columns were studied based on the existing calculation method for reinforced concrete. Hence the described topic is reviewed according to the book of prof. László Palotás – The Theory of Reinforced Concrete (Palotás 1973 – [7]).

LABORATORY TESTS

Test parameters

Laboratory experiments were carried out to study the buckling behaviour of single and laminated glass columns. The specimens were tested using an Instron 5989 testing machine. The scale of the geometry of specimens (height, thickness, width) was selected on the basis of existing glass columns from international and Hungarian realized projects. Test parameters of glass specimens were the following:

Constants: test arrangement, the type of support; width of glass (80 mm); interlayer material (EVA foil with thickness of 0.38 mm); edgework; temperature (+23 ± 5 °C).
Variables: type of glass layers: HSG/ non heat-treated Float; height of specimens: 1000 mm; 920 mm; 840 mm; number of glass layers and the thickness of specimens: single layer: 8 mm; 12 mm, laminated: 4.4 mm; 6.6 mm; 8.4 mm, laminated: 4.4.4 mm; The rate of loading: 0.5 mm/min; 1 mm/min. Support: Height of fixing: 95 mm; rubber plate (Shore A 80) was used between the steel supports and the glass. Simplified designation is used to distinguish the studied specimens, these are e.g. H_2(4.4)_2_920_0.5: ~ H, F: Type of glass: H – HSG; F – non heat-treated float glass; 2(4.4): Number of glass layers ex.: 4.4 mm laminated glass; 2: The number of specimen; 920: Nominate height of specimen [mm]; 0.5: Rate of loading [mm/min]. Abbreviations are used for the float laminated glass VG and for heat-strengthened laminated glass VSG. (Nehme and Jakab and Nehme 2013 – [5]).

Experimental test set-up

The load and vertical displacement of the upper cross-head of the Instron 5989 universal testing machine were continuously measured. At three different heights the buckling displacement (horizontal displacement) of all specimens were continuously measured with HBM displacement transducers during the tests. Strains at centre point on the surface of the glass panels were measured with HBM LY11-10/120 strain gauges. The tests were carried out at a room temperature (+23 ± 5 °C). At least three specimens were tested for each testing combination. Laminated specimens were loaded until all glass layers were fractured. In total, 120 specimens were tested.
PHENOMENON OF FLEXURAL BUCKLING

Flexural Buckling in Principle

Axially loaded columns start to deform without horizontal displacement at the beginning of the loading process. Damping material (rubber min. SHORE 80) is recommended to apply between the glass and the supporting steel surfaces. Therefore, the vertical displacement contains the deformation of the glass and the damping material as well. When the compression load reaches a critical value, the buckling of the column begins. This force is called critical buckling force \( N_{cr} \). In the first stage of the buckling process the loaded element can be unloaded without visible residual deformations. It should be noted that, in case of glass single layer columns, residual deformations do not occur after the critical buckling force. Until the loading force is increased up to the critical buckling force, it is mainly a stability problem (SLS - Serviceability Limit State of the columns). After reaching the critical buckling force, post-critical stages follow. In the case of further increase of the loading force, the column reaches the Ultimate Limit State (ULS), where the risk of the whole construction failure is significant. Significant displacements can be observed between SLS and ULS, which serves as a reserve of the glass column in the post-critical stages. The critical buckling force calculated with the application of the Euler formula was used as follows:

\[
N_{cr} = \frac{\pi^2 EI}{(uL)^2} \tag{1}
\]

This formula includes physical properties of the structural element. However, the critical buckling force is sensitive to variation in the effective length factor. This factor can have different values depending on the shape of the buckled elements. Figure 2 introduces general buckled shape with the value of the effective length factor. (Jakab and Nehme and Nehme 2014 – [3]).
The effective length factor in real glass columns is about 1.0. The damping material causes displacement in the supports in case of glass, and functions like a spring in the fixings – hence the real effective length factor can be more than the value 1.0. The effective length factor varies during the loading process. Until the SLS the effective length factor approaches the value 1.0 (it functions rather like a pinned support) and after the SLS it reduces. The buckling of the specimen causes more fixity in the supports, that is, it functions rather like a fixed support.

**Grouping of the Glass Specimens**

Characteristic curves are represented as loading force vs. displacement (vertical, horizontal and deformations) diagrams to study the laboratory experimental results. Curves are categorized in three separate groups according to the experimental results. Variation can be noticed in case of loading force vs. horizontal displacement diagrams. In case of the categorization, the variables are not considered e.g.: different heights, thickness etc. The grouping depends on the stages of the loading history of the specimens. The names of stages are:

1. First stable stage;
2. Unstable stage;
3. Second stable stage.

The first Group contains all of previously mentioned stages. Specific buckling point cannot be determined in case of the second Group, the unstable stage does not appear. Only one stable stage can be observed in case of the third stage. The load histories of the different Groups are introduced in the Figure 3.

The critical buckling forces can be determined by the incremental method, where the horizontal displacement increments are studied. The distribution of the groups are shown in percentage terms in Figure 4. The second Group occurs in the most cases. (Jakab and Nehme and Nehme 2015 – [4]).
Figure 3 Horizontal displacement vs. Force diagram of the Groups.

Figure 4 Grouping of the glass columns in aspect of the buckling phenomenon.

Basic Principles of Stability

According to Palotás, the first problem is the Young’s modulus in case of reinforced concrete, because at the starting moment the diagram of $\sigma$-$\varepsilon$ is curved so it must be treated rather as a plastic state. However, this diagram of glass is a straight line, that is, the behaviour is ideally elastic. After the critical buckling force, the flat shaped column starts to buckle when one side starts to unload while the other is more loaded. The first will be the tension side, the second will be the compression side. In case of concrete the unloading and the loading will occur by different Young’s modulus, however due to the equilibrium equations the two sides have to be balanced according to Engesser-Jaszinszkij-Kármán (Figure 5.). This equation provides an upper limit of critical buckling force. (Palotás 1973 – [7])
Figure 5 Stress distribution in the middle of the specimen at the cross-section after buckling according to Engesser-Jaszinszkij-Kármán (Palotás 1973 – [7]).

\[ N_{cr, E-J-K} = \frac{\pi^2 T I}{(\nu L)^2} \]  

Equation 2 is similar to the basic Euler critical buckling force formula (1), however T is a substitute buckling Young’s modulus, which contains the loading and the unloading Young’s modulus. According to Engesser-Shanley, immediately after the buckling moment a small bending stress appears before the stress changing (unloading at the tension side). The stress distribution is shown in Figure 6, where \( \sigma_0 \) is the basic compressive stress, “a” is the width of the column, the infinitesimal bending stress is \( \Delta \sigma \) and the \( E' \) is the actual and corresponding Young’s modulus (Formula 3). This method provides a lower limit for critical buckling force. (Palotás 1973 – [7])

\[ N_{cr, E-S} = \frac{\pi^2 E' I}{(\nu L)^2} \]  

Figure 6 Stress distribution in the middle of the specimen at the cross-section after buckling according to Engesser- Shanley (Palotás 1973 – [7]).
In the case of single layer glass the Young’s modulus is the same at the loading and unloading processes, hence the formula is the same as the basic Euler formula (1). From the buckling moment the bending stress increases faster than the basic compressive stress.

**EFFECT OF THE INITIAL ECCENTRICITY**

The initial curvature of the glass columns can influence significantly the critical buckling forces according to the earlier studies of the authors (Jakab and Nehme and Nehme 2015 – [4]). Engineers cannot neglect the effect of the initial eccentricity of the structures in the design. The incidence of buckling increases with the increase in the initial eccentricity that is magnified due to the effect of loading. The reinforced concrete structures have also initial eccentricity, regardless of being pre-cast or monolithic structure. In the case of glass the float glass also contains minimal initial eccentricity, however the heat strengthening processes increase it due to the specific procedure.

The critical buckling force decreases in case of more slender (λ) reinforced concrete columns and the decreasing of the concrete quality (the strength of the applied concrete). It is justified by the different Young-module of different concretes. However the quality (or Young’s modulus) of the glass can change due to the glass imperfections and the raw materials. The Equation 4 introduces the relative initial eccentricity:

\[ m_0 = A + B \left( \frac{\lambda}{100} \right)^2 \]  

(4)

Where \( m_0 \) (\( m_0 = \frac{e_0}{k} \), where \( e_0 \) is the initial eccentricity, and \( k \) is distance of the core point) is the eccentricity of the core point. “\( B \)” is a random eccentricity from the non-slimer shape. “\( A \)” depends from the material quality and inhomogeneity. The critical buckling force barely changes in the range 60-80 of the slenderness of reinforced concrete column according to Gehler 1938. In case of glass are more slender columns for instance the specimens tested (Figure 8-9). The buckling reduction factor (\( \alpha_0 \)) is needed for the calculation:

\[ \alpha_0 = \frac{1}{1+v_0^2} \]  

(5)

Where \( v_0 \) contains the effect of the slenderness, permanent loads and the initial eccentricity. The diagram of the buckling reduction factor and slenderness is needed to simplify the calculation method for the buckling structures, where the buckling reducing factor (\( \alpha_0 \)) decreases if the initial eccentricity factor (\( m \)) increases. In the future the authors would like to determine this diagram for glass columns. (Figure 7, Palotás 1973 – [7])
CALCULATION RESULTS

Slenderness and the Critical Buckling Force

The critical buckling forces were not well determined values in each case according to earlier studies (Jakab and Nehme and Nehme 2015 – [4]). Hence the critical buckling force of the third group cannot be taken into account to compare the experimental and calculated values. For instance the authors prefer to ignore the 10 mm thick specimens in case of single layer glass.

Single Layer Float Glass

To compare the experimental results a common effective length factor must be determined. 1.0 value was chosen in the present paper. The theoretical calculation of the effective length factor is comparable in the case of single layer glass and laminated glass based on the experimental results (Nehme and Jakab and Nehme 2013 – [6]). Figure 8 indicates the slenderness of the tested glass columns vs. critical buckling force. The mean results of specimens are in same position as the results which correspond to the effective length factor value 1.0, which means that the supports are rather pinned. The nominal thicknesses are also indicated in Figures 8 and 9.

Laminated Float Glass

In the case of laminated glass, significant difference can be observed in the buckling behaviour. The laboratory experimental results of laminated glass consist of two glass layers, - thickness of 4.4; 6.6; 8.8 mm - indicating a more fixed supporting
state. Although it means the supports are stiffer, the test set-up did not vary. The changes are justified by the effects of the interlayer material. When calculating the limit curve of the critical buckling load, the effect of the interlayer material cannot taken into account and the border curve will be lower located than the experimental mean results. When the inertia decreased the slenderness increased due to the interlayer material if it is compared to the single layer glasses. In the laboratory experiments short term loading was applied (loading rate of 0.5 mm/min). In the future it is suggested to study the stress sharing effects in the case of long term loading (Figure 9).

![Figure 8](image1)  
**Figure 8** Slenderness vs. Critical buckling force in case of single layer glasses.  

![Figure 9](image2)  
**Figure 9** Slenderness vs. Critical buckling force in case of laminated glasses.

**CONCLUSIONS**

Three different stages can be distinguished in the buckling behaviour of glass columns. However, the 2nd Stage may be missing depending on the initial shape and stiffness of the supporting system. At the buckling moment the stress distribution is similar to the Engesser-Shanley theorem, that is, it is linear. The critical buckling force decreases in case of more slender columns and the decreasing of the glass quality (e.g. the strength of the applied glass). The range of the effective length factor in the reality of a glass columns varies about 1.0. Although the damping material causes displacement in the supports, the real effective length factor can be taken as 1.0. The critical buckling forces of the single layer glass are closer to the calculated values because the basic Euler formula does not calculate with the effect of interlayer foil.

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**1. REFERENCES**


